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Аппроксимация участка траектории движения в плоскости динамической управляемой системы при помощи логарифмической спирали

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Motion of a dynamic controlled plant (DCP) in a plane *XOZ* can be defined with following system of differential equations [1]

$$\begin{cases} \dot{V} = g \cdot n_X; \\ \dot{\Psi} = \left(g/V \right) \cdot n; \\ \dot{x} = V \cdot \cos \Psi; \\ \dot{z} = V \cdot \sin \Psi, \end{cases} \qquad (1)$$

where V is a velocity of DCP;

g is a gravity force;

Ψ is a trajectory rotation angle of DCP in the plane *XOZ*;

x and z are coordinates of DCP in the plane *XOZ*.

Control vector of DCP

$$\mathbf{u} = [n_X, n]^T, \; n_X^{\min} \le n_X \le n_X^{\max}, \; n^{\min} \le n \le n^{\max}, \qquad (2)$$

where n_X is a tangential g-load;

n is a normal g-load.

Initial conditions for system (1) are

$$t_0 = 0,\ x(0) = z(0) = \Psi(0) = 0,\ V(0) = V_0 > 0. \tag{3}$$

Integration of the first equation of system (1) will give following equation

$$V(t) = V_0 + g \cdot n_X \cdot t. \tag{4}$$

Using (4) when integrating the second equation of the system (1) will give

$$\Psi(t) = \frac{n \cdot \ln(V_0 + g \cdot n_X \cdot t)}{n_X} - \frac{n \cdot \ln(V_0)}{n_X}. \tag{5}$$

Let's express time t in terms of the angle Ψ in (5)

$$t(\Psi) = -\frac{-e^{\left(\frac{\Psi \cdot n_X + n \cdot \ln(V_0)}{n}\right)} + V_0}{g \cdot n_X}. \tag{6}$$

Substituting (6) to the third equation of the system (1) will give

$$\frac{dx(\Psi)}{d\Psi} = \frac{e^{2 \cdot \left(\frac{\Psi \cdot n_X + n \cdot \ln(V_0)}{n}\right)} \cos(\Psi)}{n \cdot g}. \tag{7}$$

Let's integrate (7) with zero initial conditions (3) and simplify the result

$$x(\Psi) = \frac{V_0{}^2 \cdot e^{\left(\frac{2\Psi \cdot n_X}{n}\right)} \left(2n_X \cdot \cos(\Psi) + n \cdot \sin(\Psi)\right) - 2V_0{}^2 \cdot n_X}{g \cdot (4n_X{}^2 + n^2)}. \tag{8}$$

Expression $2n_X \cdot \cos(\Psi) + n \cdot \sin(\Psi)$ in (8) can be transformed to the following one

$$\sqrt{4n_X{}^2+n^2}\cdot\left(\frac{2n_X}{\sqrt{4n_X{}^2+n^2}}\cdot\cos(\Psi)+\frac{n}{\sqrt{4n_X{}^2+n^2}}\cdot\sin(\Psi)\right)$$

Let's denote

$$\cos(\varphi)=\frac{2n_X}{\sqrt{4n_X{}^2+n^2}},\ \sin(\varphi)=\frac{n}{\sqrt{4n_X{}^2+n^2}}. \qquad (9)$$

Such denotation is admissible since $\cos^2(\varphi)+\sin^2(\varphi)=1$, then

$$\sqrt{4n_X{}^2+n^2}\cdot\left(\cos(\varphi)\cdot\cos(\Psi)+\sin(\varphi)\cdot\sin(\Psi)\right).$$

Finally with the help of formula for cosine of angles difference with a glance of (9) we will obtain the following expression

$$2n_X\cdot\cos(\Psi)+n\cdot\sin(\Psi)=\sqrt{4n_X{}^2+n^2}\cdot\cos(\Psi-\varphi). \qquad (10)$$

Substitution of (10) to (8) will give

$$x(\Psi)=\frac{V_0{}^2\cdot e^{\left(\frac{2\Psi\cdot n_X}{n}\right)}\cdot\cos(\Psi-\varphi)}{g\cdot\sqrt{4n_X{}^2+n^2}}-\frac{2V_0{}^2\cdot n_X}{g\cdot(4n_X{}^2+n^2)}. \qquad (11)$$

Equation for $z(\Psi)$ can be obtained by analogy

$$z(\Psi)=\frac{V_0{}^2\cdot e^{\left(\frac{2\Psi\cdot n_X}{n}\right)}\cdot\sin(\Psi-\varphi)}{g\cdot\sqrt{4n_X{}^2+n^2}}+\frac{V_0{}^2\cdot n}{g\cdot(4n_X{}^2+n^2)}. \qquad (12)$$

Let's show that obtained equations (11) and (12) of planar motion of DCP are the same for a logarithmic spiral. As known, the logarithmic spiral in the plane XOZ is described by the following system

$$\begin{cases} x=a\cdot e^{b\theta}\cdot\cos\theta, \\ z=a\cdot e^{b\theta}\cdot\sin\theta. \end{cases} \qquad (13)$$

Denoting by

$$a=\frac{V_0{}^2}{g\cdot\sqrt{4n_X{}^2+n^2}},\ b=\frac{2n_X}{n},\ c_x=-\frac{2V_0{}^2\cdot n_X}{g\cdot(4n_X{}^2+n^2)},\ c_z=\frac{V_0{}^2\cdot n}{g\cdot(4n_X{}^2+n^2)}, \tag{14}$$

we can get following system from equations (11) and (12)

$$\begin{cases} x(\Psi)=a\cdot e^{b\Psi}\cdot\cos(\Psi-\varphi)+c_x, \\ z(\Psi)=a\cdot e^{b\Psi}\cdot\sin(\Psi-\varphi)+c_z, \end{cases} \tag{15}$$

which are equations of logarithmic spiral turned by the angle φ and shifted along the axis *OX* by the value c_x and along the axis *OZ* by the value c_y, where $\left[c_x,\, c_y\right]$ are coordinates of the spiral origin (see fig. 1). Angle φ can be found from (9)

$$\varphi=\arcsin\left(\frac{n}{\sqrt{4n_X{}^2+n^2}}\right)=\arccos\left(\frac{2n_X}{\sqrt{4n_X{}^2+n^2}}\right)=arctg\left(\frac{1}{b}\right). \tag{16}$$

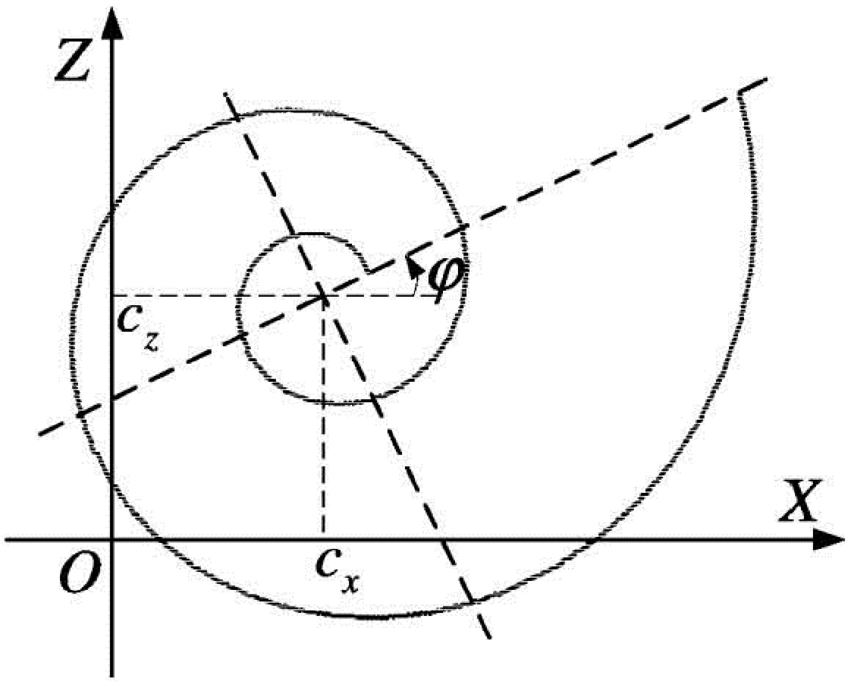

Fig. 1. Two turns of logarithmic spiral turned by the angle φ

and shifted along the axis *OX* by the value c_x and along the axis *OZ*

by the value c_y relative to the spiral origin

Hereby, planar motion of DCP described by system of differential equations (1) with initial conditions (3) and controls (2) constant on a considered time interval gives a path segment described by logarithmic spiral (15), see fig. 2.

In the case of non-zero initial conditions (3) carryover of coordinate origin to the current position and rotation of axis by the current trajectory rotation angle of DCP can be performed.

As a result there can be presented an algorithm of logarithmic spiral approximation of planar motion path segment of DCP [2].

Determination of current state $\mathbf{x}$ (t_k) of DCP at an instant time t_k

$$\mathbf{x}(t_k) = \left[V_k, \Psi_k, x_k, z_k\right].$$

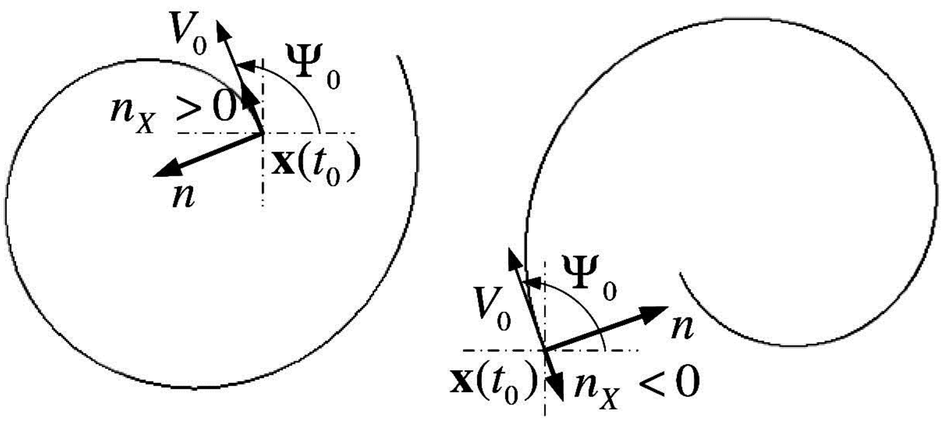

Fig. 2. Left: one turn of path segment is a distance of acceleration ($n_X > 0$);

right: one turn of path segment is a stopping segment ($n_X < 0$)

Load of DCP state at a previous instant time t_{k-1}

$$\mathbf{x}(t_{k-1}) = \left[V_{k-1}, \Psi_{k-1}, x_{k-1}, z_{k-1}\right].$$

Evaluation of current tangential g-load of DCP on the basis of (4) by the following equation

$$n_X(t_k) = \frac{V(t_k) - V(t_{k-1})}{g \cdot (t_k - t_{k-1})}. \qquad (17)$$

Evaluation of current normal g-load of DCP on the basis of (5) by the following equation

$$n(t_k) = \frac{\Psi(t_k) \cdot n_X(t_k)}{\ln \dfrac{g \cdot n_X(t_k) \cdot (t_k - t_{k-1}) + V_{k-1}}{V_{k-1}}}. \qquad (18)$$

Formulas (17) and (18) define vector of control parameters of DCP $[n_X(t_k), n(t_k)]$ at the time interval $[t_{k-1}, t_k]$. Requirement of consistency of control parameters during the time interval can be abide by the duration decrease of this time interval.

Path segment of the planar motion of DCP at each time interval $[t_{k-1}, t_k]$ can be approximated by the section of logarithmic spiral by formulas (14) – (16).

The main advantage of use of logarithmic spiral approximation of the planar motion of DCP with dynamics (1) is analytic description without need of solution of differential equations.

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Approximation of the segment of dynamically controlled system flat movement trajectory with the usage of logarithmic spiral.

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Description of dynamically controlled object flat trajectory with piecewise constant plots of control was obtained with use of logarithmic spiral plot. Examples of the segments of acceleration and deceleration with fixed values of normal and tangent overloads were included in the article. The usage of this approximation method for models moving along the flat trajectories allows to substitute system of differential equations with analytical dependences. That technique leads to significant reduce in trajectory creation time.

Publications with keywords: structural synthesis

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